

Number Theory for Algebra
Exponents
Rational Numbers
Signed Number Operations
Ratio and Proportion
Variables and Expressions
Equations and Formulas
Data and Probability
Proportional Reasoning
Patterns
The Coordinate Plane
Inequalities

Aim for Algebra
Not business as usual.

Presentation by Mardi Gale
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Interactive Format

Quick polling
Open responses
Breaks for responding to chat questions & comments
Participants on just the teleconference can email questions to:
eventquestion@wested.org

Handout

Throughout the presentation, we will be referring to a handout that can be downloaded from the SchoolsMovingUp website.

This handout will provide improved images of slides included in the Power Point presentation.

Who is in this webinar?

Site or district administrator
Teacher (classroom or resource)
Secondary mathematics teacher
EL coordinator
Community member
Higher education staff
SEA staff
Other (type in the chat area)

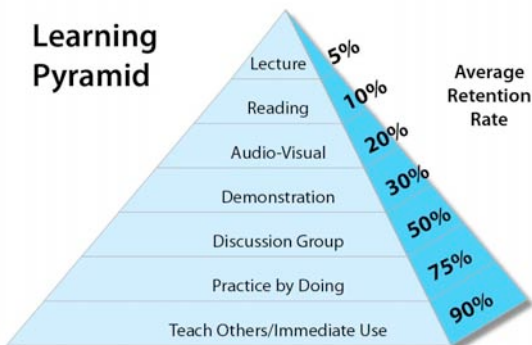


"Frank was never any good at arithmetic. He left a third of his money to me, a third to you, a third to Cindy, and a third to Matt."

High achieving students spend more time on concepts and applications and cover more topics.

Students working below grade level receive curriculum heavy on review and drill with a minimum amount of new content.

Learning Pyramid



National Training Laboratories, Bethel, Maine

Effective Universal Access Strategies

Connect to prior knowledge

- Graphic organizers
- Concept webs
- Show-and-tell
- Asking questions
- K-W-L (what you know, what you want to know, what you have learned)

Hands-on experiences

- Manipulatives
- Body language — pantomime, role play
- TPR (Total Physical Response)

Visuals

- Graphic organizer
- Modeling and demonstration
- Using physical models
- Tableau (students interpret from text)

Preview new vocabulary

- Word Wall
- Picture Dictionary

Comprehensible input

- Linking to familiar contexts
- Instructing in small groups
- Using language effectively: intonation, pauses, rephrases, repetition, simplified vocabulary, familiar vocabulary
- Use consistent language when writing word problems

Small group interaction

- Pairs/groups cooperative activities
- Peer questioning
- Jigsaw activities

Low anxiety environment

- Positive reinforcement
- Students use L1 (primary language)

Which way?



Poll: Intervention Curriculum

Does your school have an algebra intervention curriculum?

Yes or No?

Essential Elements

- Targeted, not comprehensive **CURRICULUM**
- **CONCEPTUALLY** based and standards **ALIGNED**
- **FLEXIBLE** format and structure
- Embedded **INSTRUCTION**
- Purposeful **task DEVELOPMENT**
- Precise, **academic LANGUAGE** and concrete **MODELS**
- **ASSESSMENT**, pre/post and embedded
- Facilitator guides for **instructor SUPPORT**

Aim for Algebra was developed at **WestEd** by experts in the field of mathematics education.

The program is a **coherent** set of materials, **conceptual** in nature, rather than a collection of individual worksheets on isolated topics.

Our curriculum focuses on key **“trouble spots”** in algebra that typically cause difficulty because:

- students lack prerequisites,
- students have misconceptions about the content, or
- the material is complex and students need more time and practice with the topic.

Aim for Algebra is a standards-aligned, concept-based support curriculum developed by WestEd with funding from the **U.S. Department of Education’s Institute of Education Sciences** under PR/Award #R305K040003.

Essential Elements

- Targeted, not comprehensive CURRICULUM

Modules

Available Now

Signed Number Operations

Variables and Expressions

Ratio and Proportion

Patterns

The Coordinate Plane

Inequalities

Available Soon

Number Theory for Algebra

Exponents

Rational Numbers

Equations and Formulas

Proportional Reasoning

Data and Probability

Essential Elements

- Targeted, not comprehensive CURRICULUM
- CONCEPTUALLY based and standards ALIGNED

Think about what a student might do to find this result:

$$28 \div 7 = 13$$

$28 \div 7 = 13$

Check your answer

$$\begin{array}{r} 13 \\ 7 \overline{)28} \\ \underline{-7} \\ 21 \\ \underline{-21} \\ 0 \end{array}$$

$$\begin{array}{r} 13 \\ \times 7 \\ \hline 21 \\ 70 \\ \hline 28 \end{array}$$

Check #2:

$$\begin{array}{r} (13) \\ (13) \\ (13) \\ (13) \\ (13) \\ (13) \\ + (13) \\ \hline (21) \\ + (7) \\ \hline 28 \end{array}$$

47 + 29

Add 47 and 29 mentally
(no paper and pencil).

Think about what strategy you use.

$47 + 29$

$$\begin{array}{r} 29 + 1 = 30 \\ + 47 \\ \hline 77 \\ - 1 \\ \hline 76 \end{array}$$

$$\begin{array}{r} 47 + 3 = 50 \\ 29 + 1 = 30 \\ \hline 80 \\ - 4 \\ \hline 76 \end{array}$$

$$\begin{array}{r} 40 + 20 = 60 \\ 7 + 9 = 16 \\ \hline 76 \end{array}$$

$47 + 29$

$$\begin{array}{r} 10 \\ 47 \\ + 29 \\ \hline 6 \end{array}$$

$$\begin{array}{r} 40 + 7 \\ 20 + 9 \\ \hline 60 + 16 \end{array}$$

47 - 29

Subtract 29 from 47 mentally.
Think about what strategy you use.

$$47 - 29$$

$$\begin{array}{r} \cancel{3}1 \\ \cancel{4}7 \\ - 29 \\ \hline 18 \end{array}$$

$$\begin{array}{r} 40 + 7 = 47 \\ - (20 + 9 = 29) \\ \hline 20 - 2 = 18 \end{array}$$

47 x 29

Multiply 47 and 29 mentally.
Think about what strategy you use.

$$47 \times 29$$

	40	7
20	800	140
+		
9	360	63

$$\begin{array}{r} 40 + 7 \\ \times (20 + 9) \\ \hline 800 + 63 \\ 360 + 140 \end{array}$$

$$(40 + 7)(20 + 9) = 800 + 360 + 140 + 63$$
$$(x + 3)(x + 5)$$

Poll

Do you have any questions or comments about the information presented so far?

SIGNED NUMBER OPERATIONS

SECTION	KEY CONCEPTS/INCLUDE	CCSS/STATE STANDARDS
1a	Identify and compare real numbers on number lines	HS-1.N.A.1.A.1-3 HS-1.N.A.1.A.4 HS-1.N.A.1.A.5 HS-1.N.A.1.A.6
1b	Understand density and compare absolute value	HS-1.N.A.1.A.1-3 HS-1.N.A.1.A.4 HS-1.N.A.1.A.5 HS-1.N.A.1.A.6
2a	Use real number operations to solve problems involving rational numbers	HS-1.N.A.1.B.1-3 HS-1.N.A.1.B.4 HS-1.N.A.1.B.5 HS-1.N.A.1.B.6
2b	Use real number operations to solve problems involving rational numbers	HS-1.N.A.1.B.1-3 HS-1.N.A.1.B.4 HS-1.N.A.1.B.5 HS-1.N.A.1.B.6
2c	Use real number operations to solve problems involving rational numbers	HS-1.N.A.1.B.1-3 HS-1.N.A.1.B.4 HS-1.N.A.1.B.5 HS-1.N.A.1.B.6
2d	Use real number operations to solve problems involving rational numbers	HS-1.N.A.1.B.1-3 HS-1.N.A.1.B.4 HS-1.N.A.1.B.5 HS-1.N.A.1.B.6
3a	Apply knowledge of operations of signed numbers to contextual situations and games	HS-1.N.A.1.C.1-3 HS-1.N.A.1.C.4 HS-1.N.A.1.C.5 HS-1.N.A.1.C.6

Essential Elements

- Targeted, not comprehensive CURRICULUM
- CONCEPTUALLY based and standards ALIGNED
- FLEXIBLE format and structure

Poll: Challenges

What are some of the challenges as you consider implementing an algebra intervention program?

AFTERNOON SNACKS

Ben and Aaron each bought the same items at the snack store.

Ben bought the total cost should be \$28.00. Aaron bought the total cost should be \$18.00.

Ben bought: 2 snack bars, 3 fruit drinks, 4 fruit smoothies, 2 snack bars.

Aaron bought: 3 snack bars, 4 fruit drinks, 2 fruit smoothies, 4 snack bars.

Who was correct? Explain why the answer was correct.

What was did the other person make?

Essential Elements

- Targeted, not comprehensive CURRICULUM
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- Embedded INSTRUCTION
- Purposeful task DEVELOPMENT

Seats at the Table

Maria wants to know how many people for can seat around a row of trapezoidal shaped tables.

She started by drawing the sketches below. Then she made a chart.

1 table = 8 people
 2 tables = 11 people
 3 tables = 14 people
 4 tables = 17 people

Help Maria by answering the following questions. Use what you've learned about sequences and the recursive rule.

1. Complete Maria's chart.

Number of tables in a row	1	2	3	4	5	6	7	8
Number of people seated	8							

2. Maria says that for can seat 42 people around 13 tables. Is he correct? How do you know?

3. Without drawing the tables, tell how many people can sit around 13 tables. How do you know?

Take a few minutes to work on this activity sheet in your handout packet.

Seats at the Table

Maria wants to know how many people for can seat around a row of trapezoidal shaped tables.

She started by drawing the sketches below. Then she made a chart.

1 table = 8 people
 2 tables = 11 people
 3 tables = 14 people
 4 tables = 17 people

How does the number of tables relate to the number of addends of 3?

$2 + 3t$

Number of tables

Help Maria by answering the following questions. Use what you've learned about sequences and the recursive rule.

1. Complete Maria's chart.

Number of tables in a row	1	2	3	4	5	6	7	8
Number of people seated	8							

2. Maria says that for can seat 42 people around 13 tables. Is he correct? How do you know?

3. Without drawing the tables, tell how many people can sit around 13 tables. How do you know?

Now how does the number of tables relate to the number of addends of 3?

$$5 + 3(t - 1)$$

Number of tables less 1

Seats at the Table

Maria wants to know how many people for can eat around a row of tapered shaped tables.

She started by observing the sketches below. Then she made a chart.

1 table = 5 people
 2 tables = 8 people
 3 tables = 11 people
 4 tables = 14 people

Help Maria by answering the following questions.
 Use what you've learned about sequences and the recursive rule.

1. Complete Maria's chart:

Number of tables in a row	1	2	3	4	5	6	7	8
Number of people seated	5							

2. Maria says that for can seat 42 people around 13 tables. Is he correct?
 How do you know?

3. Without drawing the tables, tell how many people can sit around 13 tables.
 How do you know?

Think of the number of people seated as the number of terms in the sequence.

Now how does the number of tables relate to the number of addends of 3?

$$4 + 4 + 3(t - 2)$$

Number of tables less 2

Seats at the Table

Maria wants to know how many people for can eat around a row of tapered shaped tables.

She started by observing the sketches below. Then she made a chart.

1 table = 4 people
 2 tables = 7 people
 3 tables = 10 people
 4 tables = 13 people

Help Maria by answering the following questions.
 Use what you've learned about sequences and the recursive rule.

1. Complete Maria's chart:

Number of tables in a row	1	2	3	4	5	6	7	8
Number of people seated	4							

2. Maria says that for can seat 42 people around 13 tables. Is he correct?
 How do you know?

3. Without drawing the tables, tell how many people can sit around 13 tables.
 How do you know?

Think of the number of people seated as the number of terms in the sequence.

Different ways students describe how the pattern grows:

$$2 + 3t \qquad 4 + 4 + 3(t - 2)$$

$$3t + 2 \qquad 8 + 3(t - 2)$$

$$5 + 3(t - 1) \qquad 5t - 2(t - 1)$$

Reconcile

The Zero Case

A useful strategy for finding the explicit rule for a pattern is to find the "zero case". The zero case is found by observing the output value when the input value is 0.

Complete the table, and then answer the 3 related questions below.

Input (x)	0	1	2	3	4
Output (y)	5	8	11	14	

1. What is the output value corresponding to an input of 0?
 How do you know?

2. What is the recursive rule for the output values?
 How do you know?

For the zero case, use the recursive rule to work backwards from the output value of 5. The input value is the input value for input value 0.

3. What is the output value for 0?
 How do you know?

No, when x (input) is 0, y (output) is 5, and when x is 1, y is 8, and so on. The output value for the zero case tells us the constant term of the explicit rule.

Now let's break the address for each input value. Using the recursive rule helps us know what input value to use in the pattern that will determine the output for any input in the pattern. We can write the table vertically to show the breakdown of the recursive rule.

Input (n)	Output (f)	Recursive Breakdown	Initial Value
0	2	$2 + 0 \times 3$	2
1	5	$2 + 1 \times 3$	2
2	8	$2 + 2 \times 3$	2
3	11	$2 + 3 \times 3$	2
4	14	$2 + 4 \times 3$	2
5	17	$2 + 5 \times 3$	2

How many addends of 3 are there for each of _____
 for each of _____
 for each of _____

2. Fill in the table above for input values 6 and 8.

3. What is the relationship between the term number, input and the number of addends of 3 for the corresponding output value?

The explicit rule for this pattern is:
 The output value is equal to 2, plus the quantity "n" times the input value."
 An equation to describe the rule is:
 $f = 2 + 3n$ or $y = 2 + 3x$ or $y = 3x + 2$

The strategy of building the staircase to help find the explicit rule is useful in identifying patterns, but it does not have meaning for some patterns that describe real objects, shapes, or situations.

Now, back to the "why" question: Why is the recursive rule "2 plus 3 times the input"?

PATTERNS

The "why" is the same as the "why" of the shaded square.

Size 1 Size 2 Size 3 Size 4

Both show the patterns above. Small white tiles surround shaded squares. The shaded squares have different side lengths. Then for each table on the right.

- Fill in the empty boxes in Bob's table.
- Bob thinks that he will need a total of 100 tiles to surround a shaded square with side length of 12 inches. Is Bob correct? How do you know?
- How many small white squares are needed to surround a shaded square with side length of 12?
- Write the rule describing how you determined the correct number of squares for size 12, in words and an algebraic equation. Refer to the geometric figures in your explanation of your rule.

Length of Side of Shaded Square (s)	Number of Small White Squares (w)
1	8
2	12
3	16
4	
5	
6	
7	

Essential Elements

- Targeted, not comprehensive CURRICULUM
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- FLEXIBLE format and structure
- Embedded INSTRUCTION
- Purposeful task DEVELOPMENT
- Precise, academic LANGUAGE and concrete MODELS

Poll:

Which is the largest number?

4 -7

Why it Works

We can use algebra to show why the number game on page 1 works for any number you choose.

Let the variable x represent any number.

Steps	Algebraic Notation
1. Pick any number	x
2. Subtract 2	$x - 2$
3. Multiply by 3 Simplify using the Distributive Property	$3(x - 2)$ $3x - 6$
4. Add 15 Simplify by combining like terms	$3x - 6 + 15$ $3x + 9$
5. Divide by 3 Simplify using the inverse of the Distributive Property	$\frac{3x + 9}{3}$ $\frac{3x}{3} + \frac{9}{3}$ $x + 3$
6. Subtract the original number Simplify	$x + 3 - x$ 3

What is your final answer?

Poll:

Precise Language

How do you see precise language and modeling reflected in the programs in your mathematics classrooms?

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- **ASSESSMENT**, pre/post and embedded

Poll:

How could you correctly answer this question without knowing what perimeter is?

◆ A typical geometry test question today:


Find the perimeter.



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♦ A problem that gives a better indication of a student's knowledge of perimeter is:

Draw a six-sided irregular polygon with a perimeter of 23 units. Show all dimensions.




Sources of Assessment Items

- Linked directly to instruction
- Includes items from national exams, such as NAEP and TIMSS
- Includes items adapted from state standard exams and state exit exams

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SEATY AT THE TABLE (ADAPTED FROM NAEP)	FACILITATOR GUIDE
<p>PURPOSE: Apply understanding of recursive rules and patterns using geometric representations.</p> <p>This task may serve as a formative assessment.</p> <p>LAUNCH: Before giving students the item to work on:</p> <ul style="list-style-type: none"> • Ask students if they can think of situations where they might need to arrange tables to maximize seating. For example, party planning, planning for family meals at Thanksgiving or during the holiday season, or event planning. • Discuss how they would divide the table arrangements. • Draw the first picture on the overhead/board or use pattern blocks to show the recursive rule of one table. • Establish the way in which people sit at the tables. That is, 1 person on each end, 1 on the short side, and 2 on the long side. • Draw model putting a second table on the overhead, or show or sketch the second picture from the task. • Discuss where people can sit so that the tables have been pushed together. Remind them that the rule for adding tables is that they connect end to end, not sides. • Read the questions together to be sure students understand the questions asked before students complete the items. • Students may work on the items independently. <p>Summary:</p> <ul style="list-style-type: none"> • Ask students to share their responses. Accept many different explanations for item 5. • Ask students to use the diagram to explain their solution. That is, explain their responses in terms of the pattern. (When another table is added, two of the ends are no longer available, but the person on one end is now sitting at the end of the new table and each time a table is added, 4 more people can sit with the groups—circle the 3 people.) • Ask students to make the connection between the diagram, the tables at the right of the diagram, and the table in item 1. (They all represent the same pattern, but are displayed in different formats.) <p>This will serve as a reflection and analysis on the work they have performed.</p>	 <p>A great set up for the situation. If possible, use the book <i>Right and Wrong</i> by all tables from the book, by Marilyn Burns and Gordon Silverman to the class. It discusses various patterns, the same and more people are seated and the table arrangements used to give the order or accommodate the number of people.</p>

EXPRESS YOUR AGE AND NOVA AND MAX

Express Your Age, Purpose: To provide practice creating a contextual example of an expression to describe a quantity.

Launch, Two or Three:

- Remind students the expression itself represents the quantity described, as an equation or variable to represent the answer is necessary.
- For example, twice 3 and 6 can be represented in several different ways and all of them represent the single quantity equal to half the sum of 6 and 6.
- Recognizing that the expression represents the quantity asked for is important for items 6 and 7 in "Nova and Max." See Summary below.
- Students will have different responses for these items. As they share their responses, ask them to share how they found the quantities. For example, if their age is 14, how did they find the answer to item 6? Ask students to represent their given with equations $14/2$ or $2(7)$.

Nova and Max, Purpose: To extend the practice of representing information or data with numbers to representing quantities with numbers and variables, thereby creating algebraic expressions.

Launch, Before or First:

- Ask students to consider that Nova may not want anyone to know how old she is so Nova is using a variable n to represent her age in the expression they are about to write.
- Remind students to use appropriate grouping symbols where necessary to show a quantity as in item 6. This is not "the quantity of a plus two." The parentheses are applied in this expression but are helpful when reading the expression. See Summary.
- Note there are no equal signs in algebraic expressions, there are none necessary for this page. The expression represents the quantity described.

Summary: It is important that students are familiar with and use the phrase "the quantity of..." to look out for grouping symbols, when reading expressions. Recognizing the expression as one quantity helps students learn how and when to use the properties of operation to simplify and/or evaluate expressions, particularly the Distributive Property. As students share their responses, be sure they use correct language in reading their expressions.

Facilitator Guide:

Item Misconceptions:

Students may try to create equations for items 6 and 7 such as $n = 10$ or $n = 10 + n = 10$. These are not expressions and introduce a second variable, both of which are not expected or necessary for the task. The expression $n + 10$ represents Nova's age in 10 years or and of half so other notation is necessary.

In the same way, " $2n + 10$ " represents the number of years until Nova is 10 years old. Another way to think about this expression is to say "the quantity $2n + 10$ is the number of years until Nova is 10 years old."

Poll:

For your student population, how might you prioritize these essential elements as you search for an intervention curriculum?

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Its About Time
www.its-about-time.com/aim/aim.html

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Next Steps

Archive:
<http://www.schoolsmovingup.net/webinars/algebra>

Feedback:
http://www.surveymonkey.com/s.aspx?sm=X4tPDEM9WgL_2fShZwpf7epQ_3d_3d