

# Math Pathways & Pitfalls: Jump-Starting Effective & Equitable Instruction

**Math**  
PATHWAYS & PITFALLS

**Math Pathways & Pitfalls:**  
Jump-Starting Effective & Equitable Instruction

WestEd

© WestEd 2009. Illustration by Federico Jordán.



**Welcome and Introductions**

- **Carne Barnett-Clarke**  
Director, Math Pathways & Pitfalls  
cbarnet@WestEd.org
- **Alma B. Ramírez**  
Co-Director, Math Pathways & Pitfalls  
aramire@WestEd.org

[www.WestEd.org/mpp](http://www.WestEd.org/mpp)

2

© WestEd 2009 WestEd.org/mpp

**Housekeeping**

**Discussion / Interactive Format**

- Quick Polling
- Open Response
- Breaks for responding to chat questions/comments  
*Those on just teleconference can email questions to:  
eventquestion@wested.org*

3

© WestEd 2009 WestEd.org/mpp

**Poll: Who is in the audience?**

- Site or District administrator
- Teacher (classroom or resource)
- Mathematics teacher or coordinator
- EL coordinator
- Higher education staff
- Technical Assistance provider
- SEA staff
- Other (type into the chat area)

4

© WestEd 2009 WestEd.org/mpp

# Math Pathways & Pitfalls: Jump-Starting Effective & Equitable Instruction

## Context

### Challenges

- Pressure to increase scores
- Assessments difficult mathematically & linguistically

### Needs

- Mathematical understanding that does not cave in to pitfalls
- Support to develop mathematical language



© WestEd 2009

WestEd.org/mpp

5

## Grade 2, CST released item, 2008

Which number sentence is an opposite number sentence for eight plus six equals fourteen?

$$8 + 6 = 14$$

- A.  $2 + 12 = 14$
- B.  $7 + 7 = 14$
- C.  $8 - 2 = 6$
- D.  $14 - 8 = 6$



© WestEd 2009

WestEd.org/mpp

6

## Grade 2, CST released item, 2008

A teacher divides a whole class into groups to work on a class project. Each group has one-sixth of all the children in the class. How many groups are there?

- A. 2
- B. 6
- C. 7
- D. 12



© WestEd 2009

WestEd.org/mpp

7

## Grade 4, CST released item, 2008

Kira owes Mark \$5, and Mark owes Kira \$7.  
Which statement means the same thing?

- A. Kira owes Mark \$2.
- B. Kira owes Mark \$12.
- C. Mark owes Kira \$2.
- D. Mark owes Kira \$12.



© WestEd 2009

WestEd.org/mpp

8

# Math Pathways & Pitfalls: Jump-Starting Effective & Equitable Instruction

## Grade 4, CST released item, 2008

The sales tax for an item is \$0.47. What is the amount of tax rounded to the nearest dime?

- A. \$0.40
- B. \$0.45
- C. \$0.47
- D. \$0.50



© WestEd 2009

WestEd.org/mpp

9

## Poll: What makes these items difficult?

- The mathematics
- The language
- Both the mathematics and the language
- Neither – the items are easy



© WestEd 2009

WestEd.org/mpp

10

## Goal of this presentation

### Share strategies to help teachers:

- Turn Pitfalls into Pathways for Learning
- Develop Mathematical Language
- Address Multiple Modalities
- Develop a Community of Learners



© WestEd 2009

WestEd.org/mpp

11

## Format of Presentation

- Research basis for strategies
- What teachers can do to embed strategies in their own lessons
- Examples of the strategies within a curriculum
- Results from embedding the strategies in a curriculum



© WestEd 2009

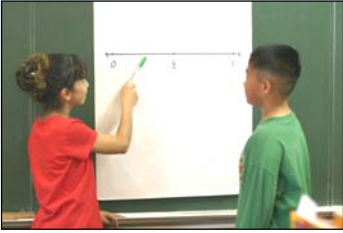
WestEd.org/mpp

12

# Math Pathways & Pitfalls: Jump-Starting Effective & Equitable Instruction

**Strategy**  
For Jump-Starting Effective & Equitable Instruction:

>>> Turn Pitfalls into Pathways for Learning



13

© WestEd, 2009. Photograph by Christian Holden. WestEd.org/mpp

**Grade 8, NAEP 2007**

Tammy scored 52 out of 57 possible points on a quiz. Which of the following is closest to the percent of the total number of points that Tammy scored?

A. 0.91%  
B. 1.10%  
C. 52%  
D. 91% (correct response)  
E. 95%

62% correct

14

© WestEd 2009 WestEd.org/mpp

**Poll**

Select a choice that can be easily eliminated if you have a fundamental understanding of percent.

Tammy scored 52 out of 57 possible points on a quiz. Which of the following is closest to the percent of the total number of points that Tammy scored?

0.91%  
 1.10%  
 52%  
 91%  
 95%

15

© WestEd 2009 WestEd.org/mpp

**Incorrect solutions are symptomatic of:**

- Fundamental misunderstandings
- Faulty reasoning
- Lack of metacognitive awareness

16

© WestEd 2009 WestEd.org/mpp

# Math Pathways & Pitfalls: Jump-Starting Effective & Equitable Instruction

## Using Pitfalls as Pathways for Learning

Tammy scored 52 out of 57 possible points on a quiz. Which of the following is closest to the percent of the total number of points that Tammy scored?

17



© WestEd 2009

WestEd.org/mpp

## Pitfalls

- Produce cognitive dissonance
- Stimulate inquiry
- Motivate metacognitive awareness

(Festinger, 1957; Borasi, 1994)

18



© WestEd 2009

WestEd.org/mpp

## Contrasting Examples: Correct vs. Pitfall

- Facilitate transfer to new problems
- Promote inferences and abstractions
- Help memory retrieval

(Gentner, Loewenstein, and Thompson, 2004; Gentner & Medina, 1998)

19



© WestEd 2009

WestEd.org/mpp

## Turning Pitfalls into Pathways for Learning In Any Lesson

### Planning a Lesson:

- Purposely select examples or problems that elicit pitfalls
- Write down common pitfalls for the problems
- Write down possible reasons for the pitfalls
- Write down questions that will elicit understanding the meaning of the problem, not just how to solve it

20



© WestEd 2009

WestEd.org/mpp

# Math Pathways & Pitfalls: Jump-Starting Effective & Equitable Instruction

## Turning Pitfalls into Pathways for Learning In Any Lesson

---

### During a Lesson:

- Discuss both correct and incorrect solutions
- Ask why a solution makes sense or doesn't make sense
- Foster an expectation that "we learn from our mistakes" and "we look out for pitfalls"



PATHWAYS · PITFALLS

© WestEd 2009

WestEd.org/mpp

21

## Turning Pitfalls into Pathways for Learning In Any Lesson

---

### On Multiple Choice Assessments:

- Discuss why particular choices are pitfalls
- Discuss which choices might make sense, and which can be eliminated
- Look back to see if the choice selected makes sense



PATHWAYS · PITFALLS

© WestEd 2009

WestEd.org/mpp

22

## Poll

---

Based on your personal experience, estimate the percentage of teachers who anticipate common pitfalls and use this information to plan effective mathematics instruction.

- 0 to 25%
- 26 to 50%
- 51% to 75%
- 76% to 100%



PATHWAYS · PITFALLS

© WestEd 2009

WestEd.org/mpp

23

## Poll

---

Do you have any questions or comments on the information just shared? Please type below.



PATHWAYS · PITFALLS

© WestEd 2009


WestEd.org/mpp

24

# Math Pathways & Pitfalls: Jump-Starting Effective & Equitable Instruction

**Strategy**  
For Jump-Starting Effective & Equitable Instruction:

**>>> Develop Mathematical Language**



25

© WestEd, 2009. Photograph by Christian Holden. WestEd.org/mpp

**Researchers Describe Academic Language**

- “The development of the spoken forms of language are essential for second language learners as a bridge to the more academic language associated with learning in school...”  
– Pauline Gibbons, *Scaffolding Language, Scaffolding Learning*, 2003
- “...mathematics ...has its own academic register features, and a considerable amount of the knowledge required for success in higher levels of mathematics is construed through language.”  
– Mary Schleppegrell, *The Language of Schooling*, 2004
- “...for ...learning to occur, students need ample opportunities to hear math language and to speak and write mathematically.”  
– Anstrom, 1999
- “Mathematics language has particular semantic and syntactic features.” – Spanos, et.al. 1988

26

© WestEd 2009 WestEd.org/mpp

**A Framework for Looking at Academic Language**

**What do we want students to be able to do?**

- **Translate** from one representational form - words, symbols, pictorial/concrete - to another
- Know the **meaning of math terms** and identify their definitions or synonyms
- **Distinguish between academic and informal** use of mathematical words, mathematical and nonmathematical use of words, and between words that sound similar, but have different meanings
- **Distill core mathematical meaning** from verbal math problems, questions, statements, or explanations even when the sentences are complex or different from everyday English
- Communicate using the **specialized academic language** of mathematics communities

27

© WestEd 2009 WestEd.org/mpp

**Helping Students Develop Mathematical Language *In Any Lesson***

**During a lesson**

- Attend to key vocabulary or “math words” that will be used authentically in the lesson.
- Limit the number of words and definitions, and provide examples and words in context.
- Develop common norms for entering into classroom conversations.

28

© WestEd 2009 WestEd.org/mpp

# Math Pathways & Pitfalls: Jump-Starting Effective & Equitable Instruction

## Helping Students Develop Mathematical Language *In Any Lesson*

- When discussing problems, use contrasting examples and solutions, including correct and “pitfall” reasoning.
- Discuss examples thoroughly and make a record of the discussion that has occurred.
- Provide opportunities for students to enter into the discussion by varying the linguistic demands of questions and of required responses.



© WestEd 2009

WestEd.org/mpp

29

## Poll

Which modality do think students in your district most often experience in the classroom?

- Visual or tactile
- Reading
- Writing
- Talking
- Listening



© WestEd 2009

WestEd.org/mpp

30

## Poll

What is your least favorite modality for learning?

- Visual or tactile
- Reading
- Writing
- Talking
- Listening



© WestEd 2009

WestEd.org/mpp

31

## Strategy

For Jump-Starting Effective & Equitable Instruction:

>>> Address Multiple Modalities



© WestEd, 2009. Photograph by Christian Holden.

WestEd.org/mpp

32

# Math Pathways & Pitfalls: Jump-Starting Effective & Equitable Instruction

## Why Multiple Modalities?

- Boosts learning
- Makes instruction more accessible to diverse learners

*(Ginns, 2005; Short, 1991)*



© WestEd 2009

WestEd.org/mpp

33

## Missed Opportunities

- U.S. emphasis on recitation style teaching was documented in TIMSS (Trends in International Mathematics and Science Study)
- Classrooms, especially with English learners, often introduce mathematics concepts with little or no discussion

*(Stigler & Heibert, 1997; Khisty, 1992)*



© WestEd 2009

WestEd.org/mpp

34

## Addressing Multiple Modalities In Any Lesson

- Ask students to **explain** and **show** their ideas with drawings or materials
- Ask students to **label** their drawings
- Ask students to **discuss** mathematical ideas
- Have them **summarize** the ideas by writing and drawing



© WestEd 2009

WestEd.org/mpp

36

## Strategy

For Jump-Starting Effective & Equitable Instruction:

### >>> Develop a Community of Learners



© WestEd, 2009. Photograph by Christian Holden

WestEd.org/mpp

36

# Math Pathways & Pitfalls: Jump-Starting Effective & Equitable Instruction

## Why a Community of Learners?

### Support for risk-taking

- There is a correlation between intellectual risk-taking and higher achievement.
- For English learners, risks are based both on their content knowledge and linguistic ability in English.
- A supportive, collaborative environment must be established that supports risk-taking.

(Beghetto, 2004; Khisty, 1995; Secada, 1992; Cohen, 1982)



© WestEd 2009

WestEd.org/mpp

37

## Developing a Community of Learners In Any Lesson

### Vary participation structures from low to high risk

- Give students time to privately grapple with ideas.
- Allow time to work with a partner to flesh out ideas, without having to expose ideas to the class until more completely formed.
- Slowly invite students to share their thinking with the entire class.



© WestEd 2009

WestEd.org/mpp

38

## Developing a Learning Community In Any Lesson

### Invite risk-taking within a respectful, cooperative learning environment

- Errors are explored for the benefit of the class, not put down or hidden.
- The entire group can collaborate to fully understand a concept by adding to or questioning others' ideas.



© WestEd 2009

WestEd.org/mpp

39

## Poll

Do you have any questions or comments on the information just shared? Please type below.



© WestEd 2009

WestEd.org/mpp

40

# Math Pathways & Pitfalls: Jump-Starting Effective & Equitable Instruction

## Poll

Do you currently use any of these strategies in your curriculum or intervention program?

- Turn pitfalls into pathways for learning
- Develop mathematical language
- Address multiple modalities
- Develop a community of learners



PATHWAYS & PITFALLS

WestEd.org/mpp

41

## Examples of these Strategies in a Math Curriculum





PATHWAYS & PITFALLS

© WestEd 2009. Illustration by Federico Jordán.

WestEd.org/mpp

42

## Embedding the Strategies Throughout a Lesson

- Math Words
- Starter Problem
- Student Thinking examples  
- Our Turn / My Turn
- Assess and Reinforce:  
Multiple Choice and Guided Writing



PATHWAYS & PITFALLS

© WestEd 2009

WestEd.org/mpp

43

## Math Words, Grade 3

**equation** An equation like  $5 + 30 = 35$  shows that 5 + 30 and 35 are equal.

**equals sign** An equals sign means "is the same amount as."

**both sides** The amounts on both sides of the equals sign in this equation are equal to 20.  
 $10 + 10 = 19 + 1$



PATHWAYS & PITFALLS

Excerpted from "What Number Is Missing?" Unit 3, Lesson 7. © 2009 WestEd, Math Pathways & Pitfalls. WestEd.org/mpp

44

# Math Pathways & Pitfalls: Jump-Starting Effective & Equitable Instruction

## Student Thinking, Grade 3

### Starter Problem

Copy and complete this equation.  
Think about the meaning.

$$20 + \underline{\hspace{2cm}} = 54$$



Excerpted from "What Number is Missing?" Unit 3, Lesson 7. © 2009 WestEd, Math Pathways & Pitfalls. WestEd.org/mpp

45

## Student Thinking, Grade 3



I read it to myself: "20 plus a missing number is equal to 54." Both sides of the equals sign need to be the same amount. The left side needs 34 more. It checks.

OK



It says to add, so 20 plus 54 is 74.

Oops!

$$20 + 74 = 54$$



Excerpted from "What Number is Missing?" Unit 3, Lesson 7. © 2009 WestEd, Math Pathways & Pitfalls. WestEd.org/mpp

46

### Starter Problem

This diagram shows that  $\frac{1}{5}$  of a school is painted. What percent is painted?

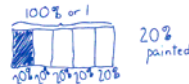


### Student Thinking



If they painted the whole school it would be 100%. So  $\frac{1}{5}$  is the same as 20% painted.

OK



$\frac{1}{5}$  is 15%. That's simple!

Pitfall

$$\frac{1}{5} = 15\%$$



Excerpted from "Relating Percent and Fractions Amounts" Unit 6, Lesson 2. © 2009 WestEd. WestEd.org/mpp

47

## Developing a Community of Learners

- Sentence stems
- Classroom posters

**Discussion Builders**

**Presenting Alternative Ideas**

- I have an idea...
- I wonder what would happen if...
- I have a counter-example...
- I have a conjecture. What if we tried...?



© WestEd 2009

WestEd.org/mpp

48

# Math Pathways & Pitfalls: Jump-Starting Effective & Equitable Instruction

## Building a Community of Learners

### Varying Risk Structures

Gradually increasing linguistic and mathematical risk structures increase intellectual risk-taking

- **Starter Problem**  
*private*
- **Read/Listen to OK and Pitfall**  
*private*
- **Discuss OK and Pitfall**  
*individual → partner → whole class*



© WestEd 2009

WestEd.org/mpp

49

## Poll

Do you have any questions or comments on the information just shared? Please type below.



© WestEd 2009

WestEd.org/mpp

50

## What is the Impact of these Strategies?

There have been two randomized trial studies conducted on the use of *MPP* lessons.

- The first study was funded by the National Science Foundation.
- The second was funded by the Institute of Education Sciences, U.S. Department of Education.



© WestEd 2009

WestEd.org/mpp

51

## NSF Research Results on the Impact of *MPP*

- Experimental research study with randomized groups
- Conducted with ethnically, linguistically, and geographically diverse student populations
- Outcome measure: mathematics learning
- Significant positive effects (ESS = 0.43 to 0.66)



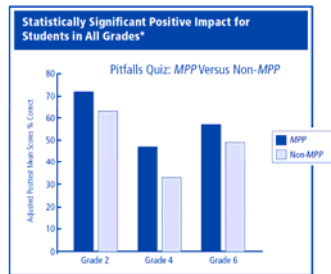
© WestEd 2009

WestEd.org/mpp

52

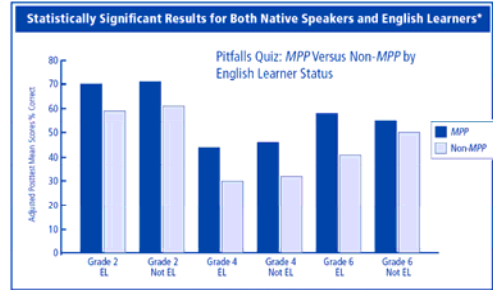
# Math Pathways & Pitfalls: Jump-Starting Effective & Equitable Instruction

## NSF Research Results: Math Learning



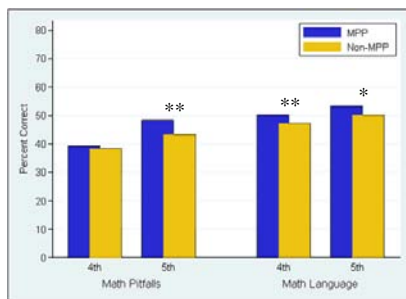
53

## NSF Research Results: English Learners



54

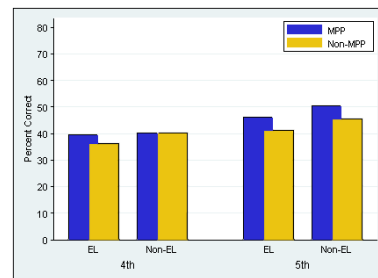
## MPP: Positive Effects on Math & Language



55

## Math Quiz

Impact of MPP for English Learners and Native English Speakers was statistically equivalent

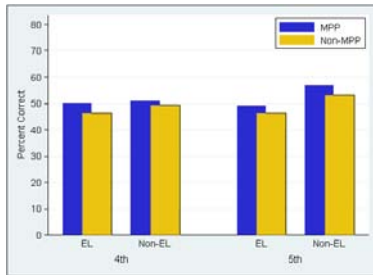


56

# Math Pathways & Pitfalls: Jump-Starting Effective & Equitable Instruction

## Language Quiz

Impact of *MPP* for English Learners and Native English Speakers was **statistically equivalent**



57



© WestEd 2009

WestEd.org/mpp

## Transfer Study: Classroom Observations

Classrooms implementing *MPP* were more likely to exhibit the following kinds of patterns and practices in post observations than pre observations during their regular (non-*MPP*) math lessons.

Mean total scores for the post-*MPP* implementation data were significantly greater than pre-implementation (Wilcoxon  $T^+ = 36$ ,  $p < .005$ ,  $n = 8$ ).

58



© WestEd 2009

WestEd.org/mpp

## Transfer Study: Classroom Observations

### Math

- More challenging and difficult problems
- Students expected to make sense of the math by sharing, showing, or explaining ideas
- Pitfalls noticed and viewed as an opportunity to learn
- Deeper understanding of the mathematical topics lesson over all

59



© WestEd 2009

WestEd.org/mpp

## Transfer Study: Classroom Observations

### Language / Discourse

- Greater variety of speaker roles and participant structures for students
- More opportunities for students to explain and justify reasoning
- Extended discussions with fellow students about mathematical ideas
- Mathematical vocabulary used with greater confidence
- Greater level of engagement in the lesson over all

60



© WestEd 2009

WestEd.org/mpp

# Math Pathways & Pitfalls: Jump-Starting Effective & Equitable Instruction

## Poll

---

Do you have any additional questions or comments on the information we have presented today? Please type below.

61



WestEd.org/mpp

## For Additional Information

---

- Visit the *Math Pathways & Pitfalls* website, [www.WestEd.org/mpp](http://www.WestEd.org/mpp)
- Or email [kdarlin@WestEd.org](mailto:kdarlin@WestEd.org)

62



© WestEd 2009

WestEd.org/mpp

## Next Steps

---

### Archive

- [www.schoolsmovingup/webinars.mpp](http://www.schoolsmovingup/webinars.mpp)

### Feedback

[www.surveymonkey.com/s.aspx?sm=jj0ICOMhrqdCodCQahxw0g\\_3d\\_3d](http://www.surveymonkey.com/s.aspx?sm=jj0ICOMhrqdCodCQahxw0g_3d_3d)

63



WestEd.org/mpp