Learning Content in the Context of Practice: A Videocase Curriculum Example

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There has been significant policy, curricular and scholarly activity around patterns, functions, and algebra in recent years, including a call by the National Council of Teachers of Mathematics and others to teach more algebra and pre-algebra content in early and middle grades, and efforts by researchers to delineate the content of early to middle-grade algebra and difficulties in its teaching (e.g., Carpenter & Franke, 2002; Stein, Baxter, & Leinhardt, 1990). Recently written curriculum materials include linear functions and algebraic representation as topics at the upper elementary and middle school grades, a change from past years, when such topics were often taught mainly in self-contained high school courses. One result of this push toward teaching algebra and pre-algebra outside of formal high school courses has been content-focused professional development for the teachers asked to add these topics to their curriculum.

One set of materials written to provide the basis for such professional development is the Video Cases in Mathematics Professional Development linear functions module (Seago & Mumme, 2002). This paper describes one effort to evaluate the effectiveness of this VCMPD module in improving teachers' understanding of mathematics – linear functions – and closely related material. Unfortunately, the sample size for this analysis is small, consisting of only eleven VCMPD participants and six comparison group teachers. With so few teachers, statistical assessments of results is impossible, eliminating the possibility of determining that any differences between groups or between pre- and post-assessments are program effects, rather than random variation. However, the results are suggestive of findings which might occur in a larger evaluation.

Below, we describe the foundations of our evaluation and analysis in the literature on teacher knowledge, and the development of measures which emerged from insights contributed by this literature. Then we describe results from this analysis, and provide a tentative discussion and conclusion.

Literature review

Policy-makers and the public have long been interested in teachers' content knowledge. Both early and current licensing exams assess prospective teachers' knowledge of the content they are to teach – history, English literature, mathematics. In most cases, such licensing exams assess what some argue is “ordinary” knowledge – historical facts and interpretation, computation and word problems in mathematics, grammar and writing craft in English. Beginning in the 1980s, however, scholars began to differentiate between such ordinary, or disciplinary knowledge of content, and the content knowledge that is used in teaching. Conceived as complementary to general pedagogical knowledge and ordinary knowledge of the content, the concept of pedagogical content knowledge was thought to include familiarity with content itself, but also thought to include familiarity with topics children find interesting or difficult, the representations most useful for teaching a specific content idea, and learners' typical errors and misconceptions (Shulman 1986; 1987; Wilson, Shulman & Richert 1987). Labeling this as “pedagogical content knowledge” not only underscored the importance of understanding subject matter in teaching, but also made plain that one's ordinary knowledge of a subject was insufficient for teaching that subject, an important new contribution to the puzzles about qualities and resources needed for effective teaching.

More recently, scholars also began to investigate and expand the “knowledge of content” category past the boundaries initially specified by Shulman and colleagues. Ball (1990), for example, describes one ability useful for teaching division of fractions – that of representing a computation problem with a real life situation, such that students can see how division of fractions might arise in the real world. Ma (1999) describes the “profound” mathematics knowledge held by some Chinese teachers, including connections between concepts and procedures, multiple perspectives on the content, deep understanding of basic ideas, and a sense of the longitudinal coherence of the K-6 curriculum. Ball and Bass (in press) describe mathematical knowledge for teaching as including the “unpacking” of mathematical ideas (also see Thompson & Thompson 1994), making connections across mathematical domains and time, and seeing mathematical themes, among other skills.

These new theoretical insights have led to renewed empirical interest in subject-specific content knowledge for teaching, with scholars describing what teachers need to know to teach English (Grossman, 1987), history (Wilson & Wineberg, 1993), and generally, across subject matters (Shulman 1986; 1987). Parallel efforts to describe teachers' content and pedagogical content knowledge in mathematics, however, uncovered a startling fact: in study after study, researchers found some U.S. teachers and pre-service teachers lacking in basic mathematical understanding, and in the ability to use mathematics in teaching (Ball, 1990; Leinhardt & Smith 1985; Ma, 1999; Thompson & Thompson 1994; Simon & Blume, 1993; Stein, Baxter & Leinhardt 1990).

In the area of linear functions, research on teacher knowledge is sparser. Stein, Baxter & Leinhardt (1990) found one fifth grade teachers' limited knowledge of functions and graphing resulted in a narrowing of instruction, including missed opportunities for making connections between concepts and representations. Sherin (1996) found Algebra I teachers' "content knowledge complexes" constrained and enabled instructional practices and shaped their interpretation of novel curriculum materials on linear functions. And Norman (1992) profiled ten experienced teachers' knowledge of functions, finding that although most teachers could provide informal and formal definitions of functions, few could connect these definitions, and some relied in inappropriate tests of functionality for complex situations. These studies, however, have only very small sample size and cover limited topics; little is known more generally about the capability of teachers at various grades in understanding linear functions in the ways necessary for teaching it.

The overall result of these empirical investigations has been policies and professional development aimed directly at improving teachers' mathematical knowledge, especially in the case of elementary and middle school teachers. Programs like the California Mathematics Professional Development Institutes are most prominent, but other efforts exist, as well. Interviews in the late 1990s with mathematics professional developers, in fact, found an almost universal belief that their product would support teachers' improved mathematics understanding, despite evidence that few sessions were promising in this regard (Hill, in press).

One challenge facing the field is how best to design learning opportunities to help teachers better their understanding of mathematical content. Conventional wisdom, at least among scholars and others who publish professional development studies and guidelines, suggests that one-shot afternoons filled with cut-and-paste mathematics activities is unlikely to assist teachers in augmenting their mathematical understanding. Instead, studies of math and science professional development suggest that this medium is likely to be successful in changing teachers' practice when it is non-traditionally organized (e.g., teacher networks or study groups), when it is extended in time, and, importantly, when it is grounded in teachers' own practice – examples of student work, curriculum materials, written cases, video clips of real-time instruction, and so forth (Carpenter et al, 1989; Cohen & Hill 2001; Garet, et al 1999; Kennedy 1999). Fewer studies of changes in teacher knowledge in mathematics exist, yet one might hypothesize, at least, that the same criteria would hold.

One promising medium for such professional development is video cases, through which teachers can observe the complexities of mathematics teaching. Video cases include several of the elements scholars argue can assist teachers (e.g., Ball & Cohen 1999): they are closely connected to real practice; they allow insight into the way mathematical ideas and tasks can be represented by various members of the classroom community (e.g., Ball & Bass in press); and they allow insight into typical student responses, difficulties, and thinking. These comprise important components of subject matter knowledge for teaching, and are significant features of the Video Cases in Mathematics Professional Development (VCMPD) linear functions module. In addition, the format of the VCMPD linear functions module allows for considerable opportunities to develop a reflective stance on instructional practice, an essential element of continual improvement in teaching (NCTM, 2000).

As with any program or professional development experience, a critical question involves what participants learn from the VCMPD linear functions module. Previous analyses (Collopy & Hill, 2002) and interviews with module developers suggest that teachers may learn, for instance, mathematical representations, and "unpacked" subject matter knowledge for teaching; ways of planning for and understanding instruction; student learning and mistakes; pedagogical strategies and student tasks; and ways of reflecting on practice. But understanding what teachers might learn is not the same as saying what participants actually learn as a result of professional development experiences, given individual differences in perception, prior knowledge, and motivation. In addition, module designers and potential users may wish to understand the relative benefits of the VCMPD package for different program goals (e.g., understanding student learning vs. pedagogy), and may also wish to compare results from VCMPD participants with participants in other programs with different designs, content, and foci (see, e.g., Kennedy 1999 for such an analysis).

In 2002, we designed a pre/post assessment for VCMPD for Fall 2002-Winter 2003 implementation. Below, we report on the construction and implementation of this assessment and on results found.

Method

Despite its name, VCMPD linear functions module is more than simply teachers viewing and discussing video of mathematics instruction. Instead, each session generally contains a careful introduction of the pedagogical and mathematical themes in each video. This includes, in many sessions, a segment on the mathematical problem itself; teachers work on the mathematical problem, then can participate in a discussion about alternative solution methods, how their solution method links to the initial problem, and so forth. Teachers view and discuss video, and in many cases, then take time to reflect on the teaching and learning in the video and link it to their own classroom practice. These eight sessions focus mainly on linear functions problems involving geometric patterns.

A previous investigation of VCMPD unit Teaching Mathematics: Developing Understanding of Linear Functions (Collopy & Hill, 2002) found that in the area of mathematics, the module focuses on topics which include multiple representations of linear functions (e.g., with symbols, geometric patterns, tables, graphs); making links between alternative representations; making links between steps in solution methods; understanding and predicting student methods for solving problems; and knowing student difficulties with linear functions. The materials focus particular attention, in the area of student understanding, on recursive vs. closed methods for representing linear functions. Accordingly, we designed an evaluation to capture teacher growth in these areas. Out of 21 total items, five focused on alternative ways to solve a problem involving geometric patterns; five focused on connections between representations; four focused on student methods, mistakes or misconceptions; seven asked teachers to solve basic linear functions problems, or problems involving other algebra content.

Overall, this evaluation consisted of 14 multiple-choice items and 7 open-ended items; of these 7 open-ended items, four can be scored correct/incorrect, and another three require in-depth analysis, owing to the variety of potential correct responses. Of the 14 multiple-choice items, ten were drawn from an item pool developed by the Study of Instructional Improvement/Learning Mathematics for Teaching (SII/LMT) projects (Ball & Hill, in preparation; Hill, Schilling & Ball 2002), and piloted with a large-N sample of elementary teachers participating in California's Mathematics Professional Development Institutes. The pre- and post-tests had identical items, as the sample sizes projected for this analysis do not allow for Rasch linking procedures.

Many of these measures are unusual, for they are not measures of "ordinary" mathematics knowledge, such as the ability to solve for x or solve a word problem, but instead measures of mathematical content as it is <u>used in teaching</u>. As Shulman (1986; 1987), Leinhardt & Smith (1985), Ball (1991) and others suggest, these items include not only those which capture "ordinary" <u>subject matter knowledge</u> (e.g., finding the eighty-third shape in a sequence), but also items which grow from the particular ways mathematics arises in elementary classrooms, or what some call <u>subject matter used in teaching</u> (e.g., examining or connecting alternative representations, analyzing student mistakes) (Ball & Bass, in press). We found it useful to illustrate this distinction by anticipating how someone who has not taught children but who is otherwise an expert in "adult" mathematics might experience these items. This test population would not find the items which tap ordinary subject matter knowledge difficult. By contrast, however, the mathematics experts might be surprised, slowed, or even halted by the mathematics-as-used-in-teaching-items; they would not have had access to or experience with opportunities to see, learn about, unpack and understand mathematics as it is used at to teach linear functions. For instance, one prominent research mathematician, upon reviewing an item used in this evaluation, expressed surprise at the variety of different methods students might use to represent a particular geometric pattern.

Items borrowed from SII/LMT were selected on the basis of their alignment with VCMPD content (e.g., representations, geometric patterns, and student thinking items were prioritized) and for their difficulty, since a previous pilot had shown many of VCMPD's middle school teachers performed quite well on the SII/LMT items, which were written primarily for elementary teachers. However, items borrowed from SII/LMT were not designed specifically for the VCMPD evaluation, and thus do not align completely with VCMPD content. This has both costs and benefits. One benefit is the measurement of VCMPD results on items written to represent, more generally, important pre-algebra knowledge for teaching. Because of the lack of complete alignment, however, the possibility for null findings increases.

Because of concerns for the security of the SII/LMT assessment, we cannot release the items used here. However, Table 1 provides some grounding in the content of this assessment: The mathematical category names the way in which the problem treats linear functions; the teaching category names any skills beyond “ordinary” knowledge that might be invoked by the problem.

In addition to developing teachers' mathematical knowledge, the module aims to foster pedagogical content knowledge for teaching linear functions and habits of reflection on practice. In particular, Collopy and Hill (2002) found that the VCMPD unit Teaching Mathematics: Developing Understanding of Linear Functions focused on anticipating and preparing for students’ strategies, methods, solutions, conceptions and misconceptions; understanding how students express their mathematical thinking during instruction; and the habit of providing evidence for supporting claims about teaching and students' understanding. In addition, the sessions have the potential for modeling an instructional approach that includes learning mathematics through generating and comparing alternative problem solving methods and solutions. Norms of reflective practice through discussions with other teachers and individually through record keeping and writing may also be modeled during the VCMPD sessions.

In order to investigate whether the module’s opportunities to learn transfer to teachers’ own preparation and enactment of instruction and their reflective practice, we designed thirteen four-point likert-type items to evaluate changes in teachers’ self-reports of pedagogy and reflective practice. The items asked teachers to think about how they had planned for teaching a particular class, taught a particular class, or engaged in reflective practices during the previous six weeks. Three constructs related to pedagogy were measured with scales consisting of three items each. Three reflective practice constructs were measured; two with single items and one with a two-item scale. Because of the limited number of subjects in this study, all scales were built theoretically. (See Table 2).

Scales were created by averaging participants’ responses across items. Responses were only included if an item was answered both on pre- and post-test. Data for all likert, multiple-choice and correct/incorrect open-ended questions were entered into SPSS for analysis of over-time performance.

In addition, teachers were asked to rate which of five activities they find most important to their planning as they prepared for class. Choices included two planning activities which we suspect are common: reading a teachers' guide and reviewing how [the teacher] taught a topic in the past. We also offered three choices that the VCMPD module may encourage: predicting different ways students might solve the assigned problems, predicting how a student might solve a problem using recursive methods, and predicting difficulties students might have with the tasks or problems in the lesson.

To capture for growth which might result from teachers learning this material as they teach in their own classrooms, as a result of learning content from the assessment itself, or from learning how to take the assessment more efficiently, the items were also administered to a comparison group of teachers not enrolled in the VCMPD program. Teachers in both groups were volunteers, and were paid comparable amounts for their participation. The instruments were administered at the beginning and end of participants' involvement in the Fall 2002 module pilot, or, for the comparison group, at corresponding time periods.

Results from our analysis are described below.

Analyses

Here, we report results from the 11 individuals enrolled in the VCMPD program and 7 comparison group members. For clarity, the discussion below is organized by item format.

Mathematics content

<u>Multiple choice items</u>. Teachers in both groups performed well on most pre-test items. Both comparison and VCMPD pilot participant groups answered 11 out of 14 multiple choice items correctly, on average (see Table 3). A slight difference is evident in the number of open-ended items answered correctly, as also shown in Table 3, with comparison group teachers more likely to correctly answer these problems. An analysis of items indicates teachers were most capable in multiple choice problems, particularly in the areas of evaluating alternative solutions to problems, linking representations, and solving other patterns, functions, or algebra problems for themselves. They

were less able in the areas of representing linear functions involving percentages, and in all non-multiple choice problems, including representing geometric patterns using linear functions, connecting solutions to geometric patterns, and evaluating student work. However, even the most difficult items were generally answered correctly by over 50% of both the comparison and VC groups. This foreshadows one problem with this evaluation – it is difficult to find evidence of growth when pre-test performance is quite high.

Because these items were linked to the SII K-6 pilot, we could compare initial status of VC and comparison group teachers to that of a broader sample. On average, both VC and comparison group teachers did much better on pre-test items than SII pilot participants, scoring roughly 80% on these pre-test items as compared to SII's pre-test average of 55%. This may be due to differences in grade level between these two groups; the SII teachers were composed of all grades K-6, where VCMPD and comparison group teachers were mainly grades 5-8.

Table 3 also shows the post-test results for both VCMPD and comparison groups. For multiple-choice items, little improvement was apparent. Neither the VCMPD nor comparison group gained substantively significant amounts on this item set as a whole; although the VCMPD group did gain roughly a half-problem, it is difficult to tell, given the sample size, whether this is statistically significant. In substantive terms, however, one-half item out of 14 total multiple choice items is a small gain.

These overall estimates, however, might obscure patterns among types of items. An inspection of individual multiple-choice items suggests that VCMPD teachers did slightly worse on those which called for the evaluation of alternative solutions to one problem, no different on items which called for the modeling of a story-problem and a linear function involving percentages, and better on a problem involving linkages between multiple representations (e.g., story, geometric, symbolic).

Constructed response items. Teachers initially performed poorly on the four constructed response items that could be simply scored as correct/incorrect. Both comparison group and VCMPD pre-tests showed teachers answering roughly two out of these four correctly. VCMPD teachers improved their performance on these constructed response items, scoring an item-and-a-half better on the post-test.

Growth in the comparison group was not as large. With such a small sample size, these results are suggestive but not definitive.

Looking at the VCMPD group's pre/post performance on each item reveals some patterns. In the case of a problem modeling 3t+1 using a geometric representation, the improvement was quite marked. While only half the teachers had solved this problem correctly in the initial assessment, all solved it correctly on the post-test. In another case, a geometric pattern that modeled 2n-1, about 3/5 of the sample answered correctly on both the pre-test and post-test. No changes were observed among comparison group teachers for either of these items or their component parts.

For both the 3t+1 and 2n-1 questions, teachers were asked how they solved the problem – making a table, graph, seeing a solution in the picture, or another method. It was hypothesized that given the VCMPD focus on multiple solution methods, teachers might use tables (a standard method for solving this problem) less frequently and other methods more frequently; alternatively, teachers might simply use more methods, regardless of approach selected. This was not the case – in fact, teachers reported more reliance on tables to solve this problem by the end of the 8-week session, and used slightly fewer methods overall. Table III shows the average number of solution methods per person for both the pre-test and post-test. In both cases, the number drops, and no differences obtain between groups.

VCMPD participants also did markedly better on an item assessing their ability to identify a common student solution to linear function problems. Figure 1 below shows the question:

Figure 1

7. Ms. Hernandez's class was looking at the following table one day. She asked her students to think of a way that would help them find *x* for any given *n*, without having to continue the table.

n	*x*
1	1
2	3
3	5
4	7

One student wrote "The answer is x + 2." Mrs. Hernandez was about to mark this student's work as wrong when Mrs. Johnson, her student teacher, said she had a guess about what the student might have been thinking. In <u>your</u> opinion, what might this student have been thinking?

An analysis of <u>pre-test</u> answers to this problem indicated a majority of both VCMPD and comparison group teachers could correctly identify the students' thinking as taking the recursive form, or describing the number added to the previous iteration to arrive at an answer. "x," in this case, is the previous total, rather than the number of frames, groups, or iterations. However, all VCMPD teachers who incorrectly answered this item on the pre-test answered this item correctly on the post-test.[1] One teacher, for instance, wrote the following on his/her pre-test:

n=x
n=x+1 Matches the "n" side in the table. The student could
n=x+2 have taken a look at n=1 and looked at x=3 then
n=x+3 wrote the "x" instead of the "n."
n=x+4

Although plausible, this is not a likely reason for this student's error. On the post-test, however, this same teacher reported thinking about this problem in a different way:

[1] One VCMPD teacher answered this problem correctly on the pre-test but not on the post-test.

X		If you look only at the x side of the table you can add the upper number with 2 and come up with the number below it.
1	1+2=3	
3	3+2=5	
5	5+2=7	
7	7+2=9	

While this teacher does not mention the recursive method by name, she can now identify the most likely reason for this students' answer: that the student was looking "only at the x side of the table," and saw the +2 pattern there.

Two more constructed response items asked teachers to identify mistakes students might make as they represented geometric patterns with symbols and numbers. The universe of potential student mistakes is large, so answers were coded for the presence of any hint at recursive solutions. VCMPD materials focus heavily on recursive solutions. Very few teachers, however, identified recursive solutions as a potential student mistake; there were negligible differences between groups, and between the pre- and post-test. This may be due, however, to the wording of this problem. Teachers (and others) may not view recursive solutions as a student mistake.

Finally, four more constructed response items, focusing on choosing sequences of geometric shapes to begin a unit on linear functions, were thrown out of this analysis. These items were designed to assess whether teachers would avoid introducing the unit with geometric patterns whose slope = 1 (potentially difficult for students, as the slope becomes invisible) or whose slope = intercept (potentially difficult because it obscures how each is shown in the pattern; see VCMPD Facilitator Guide, p. 5). However, very few teachers on either the pre- or post-tests rejected patterns constructed to represent these situations. In most cases, teachers objected to patterns based on their perceived complexity or the presence of shared sides. It is difficult to tell why these items failed; it could be the contextualization of the equations in geometric patterns obscured the problems with $y=x$ and $y=2x+2$. In some cases, it was apparent that teachers' lack of content knowledge impeded their ability to choose patterns for use at the beginning of a unit on linear functions; one teacher rejected many options and said she would use something simple like a "staircase" function (which is in fact quadratic); another teacher claimed that the number of blocks added for one figure was not constant between frames, although it clearly was.

Pedagogical and planning practices VCMPD and comparison teachers' pretest responses on the pedagogy and planning items were remarkably similar (Table 4). Differences between the group averages on the three pedagogy scales ranged from .186 to .345 on a four-point likert-type scale. Further, the highest non-response rates on the pretest were to items referring to "recursive methods," suggesting that teachers may not have understood the term. Four teachers (22% of the sample) did not respond to the item asking how often they had their students compare recursive and closed solutions to a particular problem. Three teachers (16% of the sample) did not respond to the item asking how often they prepared for class by predicting how a student might solve a problem using recursive methods.

VC and comparison group scores on all pedagogy and planning scales were nearly stable from the pretest to the posttest[2]. All but one change on the likert items was measured in hundredths of a point. The largest pre/posttest change was a *drop* within the VCMPD group of 0.316 points on having students generate and compare alternative solutions. This might indicate some teachers had a greater understanding of what it might mean to have students generate and compare alternative solutions. Little, if any, time in the VCMPD sessions is devoted to developing lessons and curriculum for teachers' own classrooms. However, participants spent considerable time generating and comparing alternative solutions to mathematics problems. Participants also view and discuss videoclips of students explaining alternative problem solving. Their own experience and the video models may have given teachers enhanced understandings of what could be happening (and isn't) in their own classrooms.

Teachers' reports of the activity they find most important to their planning were very similar across the groups on both the pre-test and post-test (see table 5). The majority of teachers in both groups chose predicting difficulties students might have with the tasks or problems in the lesson. A minority of teachers in each group chose "reading a teachers' guide" or "reviewing how I had taught a topic in the past." No teachers in either group chose "predicting different ways students might solve the assigned problems or predicting how a student might solve a problem using recursive methods."

[2] We also did an item by item analysis which yielded similar results.

Reflective practice measures On the pre-test, VCMPD teachers on average reported reflecting on their teaching with other teachers slightly more frequently than teachers in the comparison group. Comparison group teachers reported keeping detailed records of their teaching and reflecting on their teaching in writing slightly more often than the VCMPD teachers.

In contrast, the groups did differ on their reports of how often they referred to evidence from their teaching to answer a question. On average, VCMPD teachers responded that they referred to evidence a few times a month (2.0), while the comparison teachers reported doing so a few times a week (3.143). This is the largest difference between the VC and comparison groups on the pretest pedagogy and reflective practice scales or on any of the individual pretest likert-type items.

For both groups of teachers' self-reports of individual written reflection on teaching and reflection through discussion with other teachers remained fairly stable on the post-test. In contrast, VCMPD teachers reported a substantial increase in their reference to evidence to answer questions about teaching. The comparison teachers reported a small drop on this item. (See table 6).

Discussion & conclusion

This analysis suggests tentative optimism regarding the efficacy of the VCMPD approach in improving a teachers' capability in understanding a narrow range of subject matter in teaching linear functions. Teachers in the VCMPD group did improve their ability to algebraically represent at least one problem involving geometric patterns, connect their algebraic representation to the geometric pattern, and in another case, compare and link alternative representations of the same linear function. They were also more able to identify one potential student misunderstanding, using a recursive method for predicting the next term.

Although finding this effect of VCMPD participation, we cannot ascertain the extent to which teachers' growth between the pre/post-tests is statistically significant, given the small sample size. However, equivalent growth did not appear in the small comparison group, which provides some assurance that the improvement did not result from retaking the same items over a relatively small time span.

Although this is a small study, teachers' performance on the mathematics content items included can be analyzed for patterns. To start, improvement was most evident on the open-ended items. Lack of improvement on multiple-choice items might result from a true lack of learning, high initial performance, or lack of fit between all multiple choice items and the professional development, or any of these three in combination.

Another explanation for the overall pattern of results focuses again on the alignment of the evaluation assessment with VCMPD content. Teachers did not gain on the topics not directly covered by this module, e.g., linking symbolic and story problems which use linear algebra to model percentage change; an open-ended problem in which a geometric figure models y=2n-1. Neither area was directly covered by the VCMPD module. Common sense suggests that in the case of the linear algebra model for percentage change, this null result occurs simply because the percent/ratio element adds an additional layer of difficulty (see Post, Harel, Behr, & Lesh, 1988; Simon & Blume, 1994; Sowder et al., 1998 for discussions of teachers' difficulties with percent/proportional reasoning). However, the geometric representation of 2n - 1 tracks VCMPD content to a great extent; the main difference is that most VCMPD tasks modeled situations with a positive y-intercept, e.g., 4x + 4. The lack of transfer over this relatively small mathematical terrain is disappointing.

Analysis of teachers' self-reports of preparation for and enactment of instruction and of reflective practice is not encouraging. Neither the VCMPD nor the comparison group teachers, by and large, reported more of the practices supported by the VCMPD module. The only exception was in teachers' responses to the item asking how frequently they "referred to evidence from teaching (e.g., student work) to answer a question," where only VCMPD teachers increased their reports of this practice. There are several possible explanations for lack of change in pedagogical and reflective practice reported by the VCMPD participants. First, these items may not reliably or validly measure the underlying hypothesized constructs about pedagogy and reflective practice. Second, the VC group's self-rating after the professional development may reflect a better understanding of the language of the survey items. Teachers who better understand what "generate and compare alternative problem solving solutions during instruction" means may be less likely to report they engage in it.

Third, change takes time and resources in addition to increased knowledge and awareness. It is possible that the teachers did not have sufficient time or opportunities to incorporate what they learned in the professional development into their mathematics teaching. They also may not have had curricular materials that support the approach to teaching modeled in the VCMPD sessions. Reflecting on practice in writing and through discussions also requires resources of time and willing and able colleagues. In contrast, referring to evidence -- the area of greatest gain for the VC group -- does not require context-dependent resources.

Fourth, previous analysis of the module's opportunities to learn indicated the module was underdeveloped in several areas that would promote transfer of teachers learning about pedagogy (Collopy and Hill, 2002). Teachers may be developing declarative knowledge of pedagogical approaches without parallel procedural knowledge. Little time is devoted to developing connections between approaches modeled and viewed in the workshops and how to enact the same approach in teachers' own classrooms. For example, module participants have many opportunities to identify the potential goals of mathematical tasks and analyze the potential purposes of teachers' questions. However, they have few or no opportunities to apply these new skills by beginning with learning objectives, as teachers usually do in their own classrooms, and then developing or adapting corresponding instructional tasks. The module also provides opportunities to decode student explanations of mathematical thinking. However, it hardly touches on how to use understandings to choose and build on individual students' ideas to further the understanding of all students about a mathematical topic or to move the class into new mathematical territory. Similarly, the facilitators' guide emphasizes the importance of establishing community norms and expectations for the tone and structure of interactions and purpose of the sessions. How teachers might establish a culture that promotes inquiry in their own classroom into mathematics is not a focus of the module.

We should caution readers that this study represents only one enactment of the module. Continued development and piloting work may change results in any future evaluation. So would, we believe, the enactment of this module by facilitators with different skills, emphases, and who are working with different teacher populations.

A final note: these results apply to the VCMPD approach as a package, not to the use of video cases more generally. Module developers designed significant opportunities for teachers to learn mathematics during these sessions, including solving problems for themselves, listening to other teachers explain alternative solution methods, connecting solution methods to one another, and more. Because these sessions mixed mathematically rich discussions with showing of and discussions about video of classroom instruction, it is difficult to disentangle the effects of both on teachers' knowledge of mathematics and students. For this reason, these results cannot endorse the use of video cases generally, but only in this specific instance.

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Table 1: Content of Assessment

Item Number(s)	Mathematical Content	Teaching Content
1-4	Linear function represented as geometric pattern	Evaluating correctness of different student solution methods
5	Minimizing perimeter	--
6	Sequence	--
7-11	Linear function involving percentages	--
12-16	Linear function in geometric pattern, word problem, symbols	Connecting alternative representations of same linear function
17,18	Linear function represented as geometric pattern, writing symbolic representation to match	Making connections between representations
19	Recursive vs. closed method for naming linear functions	Understanding student work
20, 21	Linear function represented as geometric pattern, writing symbolic representation to match	--

Table 2: Pedagogy and Reflective Practice Scales and Items

Construct	Likert items
Pedagogy - Using knowledge of students' strategies, conceptions and misconceptions to plan for instruction	Thinking about how you have planned to teach your target class over the past six weeks, how often have you engaged in the following activities? • Prepared for class by predicting different ways students might solve the assigned problems • Prepared for class by predicting how a student might solve a problem using recursive methods • Prepared for class by predicting difficulties students might have with the tasks or problems in the lesson
Pedagogy - Building on students' mathematical thinking during instruction	Thinking about how you have planned to teach your target class over the past six weeks, how often have you engaged in the following activities? • Had a student(s) explain a correct answer to a problem • Modified my lesson plan to focus more attention than I'd planned on a particular student's method for solving a problem • Had a student(s) explain an incorrect answer to a problem
Pedagogy - During instruction having students generate and compare alternative solutions	Thinking about how you have planned to teach your target class over the past six weeks, how often have you engaged in the following activities? • Asked students to generate alternative methods for solving problems • Compared alternative methods for solving problems • Compared recursive and closed solutions to a particular problem
Reflective Practice - Providing evidence to support claims	Outside of the Videocase experience, how often in the past six weeks have you: • Referred to evidence from teaching (e.g., student work) to answer a question
Reflection Practice - Discussing conjectures and evidence with other teachers	Outside of the Videocase experience, how often in the past six weeks have you: • Reflected on my own teaching through discussions with another teacher

Reflection Practice - Keeping records of and referring back to thinking	Outside of the Videocase experience, how often in the past six weeks have you: • Reflected on my teaching in writing • Kept detailed records of my teaching

Table 3: Mathematics Knowledge Item Results

	Videocase	Comparison
Pre-test, multiple choice	79% (11.1/14)	78% (10.9/14)
Post-test, multiple choice	83% (11.6/14)	76% (10.6/14)
Pre-test, open-ended items	1.9/4	2.3/4
Post-test, open-ended items	3.4/4	2.7/4
Number solution methods checked, pre-test, Q17	1.83	1.83
Number solution methods checked, post-test, Q17	1.55	1.71

Table 4: Results of Pretest and Posttest Pedagogy Response of VCMPD and Comparison Teachers

Scale/ item	VCMPD			Comparison		
Pedagogy	Pre	Post	Change	Pre	Post	Change
Anticipating and preparing for students' strategies, methods and solutions, conceptions & misconceptions	3.019	3.000	-0.019	3.238	3.238	0.000
Understanding how students express themselves mathematically during instruction / deciphering individual students' math statements	3.067	3.133	0.066	2.881	2.905	0.024
Generating and comparing alternative solutions	3.083	2.767	-0.316	2.738	2.667	-0.071

Table 5: Preparing for teaching

	VCMPD		Comparison	
	Pre	Post	Pre	Post
Reading a teachers' guide	2	2	1	1
Reviewing how I taught a topic in the past	2	2	1	0
Predicting different ways students might solve the assigned problems	0	0	0	1
Predicting how a student might solve a problem using recursive methods	0	0	0	0
Predicting difficulties students might have with the tasks or problems in the lesson	5	5	5	5

Table 6: Results of Pretest and Posttest Reflective Practice Responses of VCMPD and Comparison Teachers

Scale/ item	VCMPD			Comparison		
Reflective practice	Pre	Post	Change	Pre	Post	Change
Providing evidence to support claims	2.000	3.111	1.111	3.143	2.857	-0.286
Norms of discussing conjectures and evidence with other teachers	3.222	3.111	-0.111	2.857	2.857	0.0
Norms of keeping records of and referring back to thinking	2.167	2.389	0.222	2.643	2.857	0.214